

# 14. SOAL-SOAL LIMIT FUNGSI

EBTANAS2000

1.  $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 3x + 2} = \dots$

- A. -1    B.  $-\frac{1}{3}$     C. 0    D. 1    E. -5

jawab:

kalau dimasukkan nilai  $x=2$  didapat hasil =  $\frac{0}{0}$

Gunakan L'HOSPITAL

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 3x + 2} &= \lim_{x \rightarrow 2} \frac{2x - 5}{2x - 3} \\ &= \frac{2 \cdot 2 - 5}{2 \cdot 2 - 3} = \frac{-1}{1} = -1 \end{aligned}$$

jawabannya adalah A

UMPTN2000

2. Jika  $f(x) = \frac{x^2 - 2x}{x^2 - 4}$ , maka  $\lim_{x \rightarrow 2} f(x) = \dots$

- A. 0    B.  $\sim$     C. -2    D.  $\frac{1}{2}$     E. 2

jawab:

Cara 1 : dengan L'HOSPITAL

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{2x - 2}{2x} \\ &= \frac{2 \cdot 2 - 2}{2 \cdot 2} = \frac{1}{2} \end{aligned}$$

Cara 2 : pemfaktoran

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{x(x - 2)}{(x - 2)(x + 2)} \\ &= \lim_{x \rightarrow 2} \frac{x}{x + 2} \\ &= \frac{x}{x + 2} = \frac{2}{2 + 2} = \frac{1}{2} \end{aligned}$$

jawabannya adalah D

UAN2006

3.  $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - \sqrt{2x+1}}{x-4} = \dots$

- A.  $-\frac{1}{6}$     B.  $-\frac{1}{12}$     C. 0    D.  $\frac{1}{12}$     E.  $\frac{1}{6}$

jawab:

$$\begin{aligned} \lim_{x \rightarrow 4} \frac{\sqrt{x+5} - \sqrt{2x+1}}{x-4} &= \lim_{x \rightarrow 4} \frac{\sqrt{x+5} - \sqrt{2x+1}}{x-4} \cdot \frac{\sqrt{x+5} + \sqrt{2x+1}}{\sqrt{x+5} + \sqrt{2x+1}} \\ &= \lim_{x \rightarrow 4} \frac{x+5 - (2x+1)}{(x-4)(\sqrt{x+5} + \sqrt{2x+1})} \\ &= \lim_{x \rightarrow 4} \frac{-x+4}{(x-4)(\sqrt{x+5} + \sqrt{2x+1})} \\ &= \lim_{x \rightarrow 4} \frac{-(x-4)}{(x-4)(\sqrt{x+5} + \sqrt{2x+1})} \\ &= \lim_{x \rightarrow 4} \frac{-1}{(\sqrt{x+5} + \sqrt{2x+1})} \\ &= \frac{-1}{(\sqrt{4+5} + \sqrt{2 \cdot 4 + 1})} = -\frac{1}{3+3} = -\frac{1}{6} \end{aligned}$$

UN2007

4. Nilai  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{4 - \sqrt{5x+1}} = \dots$

- A. -8    B. -6    C. 6    D. 8    E.  $\sim$

jawab:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{4 - \sqrt{5x+1}} &= \lim_{x \rightarrow 3} \frac{x^2 - x - 6}{4 - \sqrt{5x+1}} \cdot \frac{4 + \sqrt{5x+1}}{4 + \sqrt{5x+1}} \\ &= \lim_{x \rightarrow 3} \frac{(x^2 - x - 6)(4 + \sqrt{5x+1})}{16 - (5x+1)} \end{aligned}$$

$$= \lim_{x \rightarrow 3} \frac{(x^2 - x - 6)(4 + \sqrt{5x+1})}{16 - (5x+1)}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)(x+2)(4 + \sqrt{5x+1})}{15 - 5x}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)(x+2)(4 + \sqrt{5x+1})}{-5(x-3)}$$

$$= \lim_{x \rightarrow 3} \frac{(x+2)(4 + \sqrt{5x+1})}{-5}$$

$$= \frac{(3+2)(4 + \sqrt{5 \cdot 3 + 1})}{-5}$$

$$= \frac{5(4+4)}{-5} = \frac{40}{-5} = -8$$

UAN2005

5. Nilai dari  $\lim_{x \rightarrow 0} \frac{\tan 2x \cos 8x - \tan 2x}{16x^3} = \dots$

- A. -4 B. -6 C. -8 D. -16 E. -32

jawab:

$$\lim_{x \rightarrow 0} \frac{\tan 2x \cos 8x - \tan 2x}{16x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\tan 2x(\cos 8x - 1)}{16x^3}$$

$$\begin{aligned} \cos 8x &= \cos(4x+4x) \\ &= \cos 4x \cdot \cos 4x - \sin 4x \cdot \sin 4x \\ &= \cos^2 4x - \sin^2 4x \\ &= 1 - \sin^2 4x - \sin^2 4x \\ &= 1 - 2 \sin^2 4x \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\tan 2x(\cos 8x - 1)}{16x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\tan 2x(1 - 2 \sin^2 4x - 1)}{16x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\tan 2x(-2 \sin^2 4x)}{16x^3}$$

$$= \lim_{x \rightarrow 0} -2 \frac{\tan 2x}{16x} \cdot \lim_{x \rightarrow 0} \frac{\sin 4x}{x} \cdot \lim_{x \rightarrow 0} \frac{\sin 4x}{x}$$

$$= -2 \cdot \frac{2}{16} \cdot 4 \cdot 4 = -4$$

jawabannya adalah A

UN2002

6.  $\lim_{x \rightarrow 2} \frac{1 - \cos^2(x-2)}{3x^2 - 12x + 12} = \dots$

- A. 0 B.  $\frac{1}{3}$  C.  $\frac{1}{3}\sqrt{3}$  D. 1 E. 3

jawab:

$$\cos^2 x + \sin^2 x = 1 \Leftrightarrow \cos^2(x-2) + \sin^2(x-2) = 1$$

$$\Rightarrow \cos^2(x-2) = 1 - \sin^2(x-2)$$

$$\lim_{x \rightarrow 2} \frac{1 - \cos^2(x-2)}{3x^2 - 12x + 12}$$

$$= \lim_{x \rightarrow 2} \frac{1 - (1 - \sin^2(x-2))}{3x^2 - 12x + 12}$$

$$= \lim_{x \rightarrow 2} \frac{1 - 1 + \sin^2(x-2)}{3x^2 - 12x + 12}$$

$$= \lim_{x \rightarrow 2} \frac{\sin^2(x-2)}{3(x^2 - 4x + 4)}$$

$$= \lim_{x \rightarrow 2} \frac{1}{3} \frac{\sin^2(x-2)}{(x-2)^2} = \frac{1}{3}$$

Jawabannya adalah B

UAN2005

7. Nilai  $\lim_{x \rightarrow \sim} \{(3x-1) - \sqrt{9x^2 - 11x + 9}\} = \dots$

- A. -1 B. 0 C.  $\frac{1}{6}$  D.  $\frac{3}{6}$  E.  $\frac{5}{6}$

Jawab:

arahkan menjadi persamaan:

$$\lim_{x \rightarrow \infty} \left( \sqrt{ax^2 + bx + c} - \sqrt{ax^2 + px + q} \right) = \frac{b-p}{2\sqrt{a}}$$

$$\lim_{x \rightarrow \infty} \{ (3x-1) - \sqrt{9x^2 - 11x + 9} \}$$

$$= \lim_{x \rightarrow \infty} \{ \sqrt{(3x-1)^2} - \sqrt{9x^2 - 11x + 9} \}$$

$$= \lim_{x \rightarrow \infty} \{ \sqrt{9x^2 - 6x + 1} - \sqrt{9x^2 - 11x + 9} \}$$

$$= \frac{b-p}{2\sqrt{a}} = \frac{-6 - (-11)}{2\sqrt{9}} = \frac{5}{6}$$

Jawabannya adalah E

EBTANAS1994

$$8. \lim_{x \rightarrow \infty} \frac{3x-5}{2x^2+4x+5} = \dots$$

- A. 0    B.  $\frac{8}{11}$     C.  $\frac{3}{4}$     D. 1    E. 6

jawab:

rumus dasar:

$$\lim_{x \rightarrow \infty} \frac{ax^m + bx^{m-1} + \dots}{px^n + qx^{n-1} + \dots}$$

membagi pembilang dan penyebut dengan pangkat tertinggi penyebut

$$\lim_{x \rightarrow \infty} \frac{3x-5}{2x^2+4x+5} = \lim_{x \rightarrow \infty} \frac{\frac{3x}{x^2} - \frac{5}{x^2}}{\frac{2x^2}{x^2} + \frac{4x}{x^2} + \frac{5}{x^2}} =$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{3}{x} - \frac{5}{x^2}}{2 + \frac{4}{x} + \frac{5}{x^2}}$$

$$= \frac{0-0}{2+0+0} = \frac{0}{2} = 0$$

Jawabannya adalah A

UAN2006

$$9. \text{ Nilai } \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x - \sin \frac{\pi}{6}}{\frac{\pi}{6} - \frac{x}{2}} = \dots$$

- A.  $\frac{1}{2} \sqrt{3}$     B.  $\frac{1}{3} \sqrt{3}$     C.  $\sqrt{3}$     D.  $-2 \sqrt{3}$     E.  $-3 \sqrt{3}$

jawab:

Kalau nilai x dimasukkan didapat nilai:  $\frac{0}{0}$

Cara 1: L'Hospital

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x - \sin \frac{\pi}{6}}{\frac{\pi}{6} - \frac{x}{2}}$$

$$= \lim_{x \rightarrow \frac{\pi}{3}} \frac{-\sin x}{-\frac{1}{2}} = \lim_{x \rightarrow \frac{\pi}{3}} 2 \sin x$$

$$= 2 \cdot \sin \frac{\pi}{3} = 2 \cdot \frac{1}{2} \sqrt{3} = \sqrt{3}$$

Cara 2: pemfaktoran (agak panjang)  
dibahas disini sebagai perbandingan:

$$\text{Dimisalkan : } \frac{\pi}{6} - \frac{x}{2} = t$$

$$\text{maka : } \frac{x}{2} = \frac{\pi}{6} - t$$

$$x = 2 \left( \frac{\pi}{6} - t \right)$$

$$= \frac{\pi}{3} - 2t$$

$$\text{untuk nilai } x = \frac{\pi}{3} \text{ maka } t = \frac{\pi}{6} - \frac{3}{2} = \frac{\pi}{6} - \frac{\pi}{6} = 0$$

Untuk  $x = \frac{\pi}{3} - 2t$  dan  $t \rightarrow 0$ , maka

$$\begin{aligned} & \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x - \sin \frac{\pi}{6}}{\frac{\pi}{6} - \frac{x}{2}} \\ &= \lim_{t \rightarrow 0} \frac{\cos(\frac{\pi}{3} - 2t) - \frac{1}{2}}{t} \\ &= \lim_{t \rightarrow 0} \frac{\cos \frac{\pi}{3} \cos 2t + \sin \frac{\pi}{3} \sin 2t - \frac{1}{2}}{t} \\ &= \lim_{t \rightarrow 0} \frac{\frac{1}{2} \cos 2t + \frac{1}{2} \sqrt{3} \sin 2t - \frac{1}{2}}{t} \\ &= \lim_{t \rightarrow 0} \frac{\frac{1}{2}(1 - 2 \sin^2 t) + \frac{1}{2} \sqrt{3}(2 \sin t \cos t) - \frac{1}{2}}{t} \\ &= \lim_{t \rightarrow 0} \frac{\frac{1}{2} - \sin^2 t + \sqrt{3} \sin t \cos t - \frac{1}{2}}{t} \\ &= \lim_{t \rightarrow 0} \frac{-\sin^2 t + \sqrt{3} \sin t \cos t}{t} \\ &= \lim_{t \rightarrow 0} \frac{\sin t(-\sin t + \sqrt{3} \cos t)}{t} \\ &= \lim_{t \rightarrow 0} \frac{\sin t}{t} \cdot \lim_{t \rightarrow 0} (-\sin t + \sqrt{3} \cos t) \\ &= 1 \cdot (0 + \sqrt{3}) = \sqrt{3} \end{aligned}$$

Jawabannya adalah C

UAN2004

10. Nilai  $\lim_{x \rightarrow 3} \frac{(x-7) \sin(2x-6)}{x^2 + 2x - 15} = \dots$

- A. -4    B. -1    C. 0    D. 1    E. 4

jawab:

cara yang cepat dengan menggunakan L'Hospital dengan catatan kita harus menguasai differensial/turunan. Cara ini cocok untuk soal multiple choice seperti ini.

$$\lim_{x \rightarrow 3} \frac{F'(x)}{G'(x)}$$

Ingat :  $y = uv$ , maka  
 $y' = u'v + uv'$

$$F'(x) = 1 \cdot \sin(2x-6) + (x-7) \cos(2x-6) \cdot 2$$

$$G'(x) = 2x + 2$$

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{F'(x)}{G'(x)} &= \lim_{x \rightarrow 3} \frac{\sin(2x-6) + 2(x-7) \cos(2x-6)}{2x+2} \\ &= \frac{0 + 2(-4) \cdot 1}{2 \cdot 3 + 2} = \frac{-8}{8} = -1 \end{aligned}$$